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Application of Fuzzy Logic in CA/LGCA Models As a Way of Dealing With Imprecise and Vague Data

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Abstract

In this paper we discuss how to address qualitative aspects and problems of imprecise and vague data in modelling dynamics of spread of epidemics. As a possible solution we propose superposition of Fuzzy Logic with Lattice Gas Cellular Automata.

1. Introduction

The CA/LGCA ("Cellular Automata" / "Lattice Gas Cellular Automata") methodology has been used to develop and analyse models of spread of epidemics from microstructure of individual interactions, to study vaccination strategies from individual responses. H. Fukś and A. Lawniczak presented the mathematical model[1], while B. Di Stefano, H. Fukś, and A. Lawniczak described the computer implementation[2].

The mathematical model of H. Fukś and A. Lawniczak[1] was developed in response to some of the shortcomings of the "classical" methodology of differential equations, which were identified by Mollison and others[3]. In particular, it has been observed that the methodologies of differential equations do not model adequately[2]:

- Individual contact processes, through which infectious diseases are being contracted. The contact processes among individuals are fundamental for the spread of epidemics
- Effects of individual behaviour and, in particular, of changing behaviours as a result of contracting the disease or other factors
- In general, qualitative aspects of an epidemic process
- Effects of mixing patterns of the individuals involved in the spread of the epidemics, regardless of their status of "Susceptible", "Exposed", "Infectious", and "Recovered" or whatever else may be applicable
- Spatial aspects of the spread of an epidemic.

Models of epidemics using CA, capable of reducing or eliminating the limitations of the

[1] See reference item [1]

[2] See reference item [2]

[3] See reference item [3]

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"classical" methods have attracted significant attention in recent years[3].

Thus, H. Fukś and A. Lawniczak developed a model for the study of the spread of an infectious disease, based on a LGCA ("Lattice Gas Cellular Automaton"), a type of automaton akin to CA[4].

Here we describe very briefely only the automaton algorithm. The presented LGCA is an idealisation of SIR epidemic model, which is applicable to the types of epidemics involving only "Susceptible", "Infectious" and "Recovered" individuals. In the LGCA the dynamics takes place on a regular hexagonal lattice. Each node of the lattice represents some region of a physical space called a cell. The cells tile the physical space. The individuals ("Susceptible", "Infectious" and "Recovered"), represented by particles, reside in cells (nodes of the lattice) and they can move from one cell (node) to another. The time evolution of the automaton takes place at the discrete time steps. Hence, in the LGCA space, time and number of individuals are represented by discrete variables. This discretisation makes LGCA particularly suitable for computer simulations.

At each time step the following three operations interaction, randomisation and propagation are performed in synchronous. The dynamics of the automaton arises from the repetitive application of these three operations.

During the interaction operation, performed at each cell independently of the other cells, individuals residing in a cell can change their type. Each susceptible individual in the cell, independently of other individuals, can become infected with probability "$1-(1-r)^{n_I}$", where n_I is the number of infected individuals at the same cell and r is the probability of infection per contact. Hence, an individual can contract infection if there is at least one infected individual in the cell. Similarly, each infected individual independently of the other individuals can recover with a probability "a".

At the randomisation step at each cell of the lattice independently of the other cells individuals select randomly edges of the lattice which originate from the nodes representing their cells and connect to the nearest neighbour nodes. At the propagation step, individuals move from one cell to another through the selected edge of the lattice.

The described fully parallel algorithm can be easily implemented on the computer. Also, it can be easily generalised to other types of epidemic models and modified to incorporate more realistic types of motions of individuals. Like, the motion of the individuals connected to their daily activities.

The mean-field dynamics of the automaton of SIR type of epidemic can be approximated by the following set of difference equations[5]:

$$\rho_S(k+1) = \rho_S(k) - r\,\rho_S(k)\,\rho_I(k),$$
$$\rho_I(k+1) = \rho_I(k) + r\,\rho_S(k)\,\rho_I(k) - a\rho_I(k),$$
$$\rho_R(k+1) = \rho_R(k) + a\,\rho_I(k),$$

where:

k	is the k^{th} iteration
k+1	is the $(k+1)^{th}$ iteration
a	is the probability of removal
r	is the infectious rate per contact
ρ_S	is the concentration of "Susceptible"
ρ_I	is the concentration of "Infectious"

[4] For a good description of CA see reference item [3]

[5] See reference item [1], page 4, equations (7)

ρ_R is the concentration of "Removed".

H. Fukś and A. T. Lawniczak have investigated[6] effects of spatial inhomogeneitis on the dynamics of the epidemic process using examples of two vaccination strategies which only differ in spatial distribution of infectious and vaccinated individuals. They discussed the differences between mean-field dynamics and LGCA simulation results.

H. Fukś and A. T. Lawniczak have described[6] the model and have shown[6] its advantages. Indeed this model addresses some of the issues raised by Mollison and others[7].

However, there is still the problem of vague and imprecise data. This paper shows how fuzzy logic can be used to deal with imprecise and vague data, typical of the application domain under consideration.

2.1 Vague And Imprecise Data In The Study Of Epidemics

Thomas C. Quinn and Anthony S. Fauci write[8]: "By early 1997, **1.5 million** cases of AIDS had been officially reported to the World Health Organization (WHO). However, because of underreporting, reporting delays, and poor recognition of clinical cases, approximately **8.4 million** cases of AIDS are estimated to have occurred since the start of the global pandemic".

This is probably one of the most worrisome statements about the AIDS epidemic, worrisome because it shows an alarming order of magnitude for the imprecision of data. How is it possible to adequately model the spread of AIDS if the number of those infected is really unknown and could be anywhere between 1.5 and 8.4 million cases?

Scores of examples of smaller magnitude, but comparable gravity, can be given for other epidemics.

2.2 Using Fuzzy Logic

Fuzzy Logic is a form of continuous multi-valued logic allowing "computing with words"[9]. Hence, it can be applied to address qualitative aspects of epidemics.

Fuzzy logic is a useful tool when dealing with vague and imprecise data. Within the context of the present work, fuzzy logic is being used as:

- A way of translating the "application domain" expert's experience into a computer algorithm by interviewing the expert and extracting the applicable rule base
- A tool for adaptively extracting rules from the data itself.

By alternatively applying both methods in a repetitive fashion is possible to improve the results that can be obtained with one methodology only.

2.3 Fuzzy Adaptive Approximator

Fuzzy logic is a convenient way to map an input space to an output space. It is a way to model relationships of the type "cause/effect", "stimulus/response", "question/answer", etc[10].

[6] See reference item [1]

[7] See reference item [3]

[8] See reference item [5]

[9] See reference items [6] and [7]

[10] See Chapter 10, page 161, of reference item [8], that is, Bart Kosko, "Fuzzy Thinking – The New Science of Fuzzy Logic", Hyperion/Disney Books, 1993 (ISBN 0-7868-8021-X). Chapter 10, page 161.

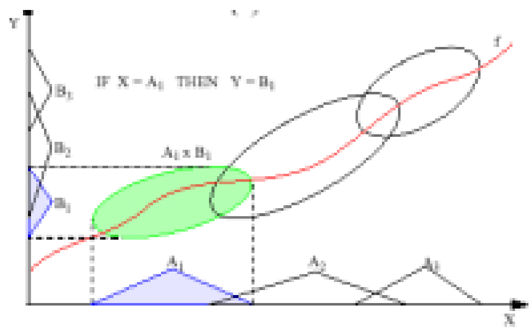

Figure 1 - Mapping Of Input Space Into Output Space[11]

Figure 1 can be used to explain the method. Let's say that there is a relationship f between an input space X (represented by the x-axis) and an output space Y (represented by the y-axis). This means that the relationship between the input space and the output space is given by y = f (x). If y = f (x) is not analytically known, we cannot write an equation in explicit form. However, we can use the relationship between the input space X and the output space Y, relationship given by Y = F(X), a relationship that links subsets of the input space X to subsets of the output space Y by means of a set (fuzzy) rules of the type:

IF X is A_x THEN Y is A_y
IF X is B_x THEN Y is B_y
… … … … … … …
IF X is Z_x THEN Y is Z_y

The labels of the type A_x in Figure 1 represents an adjective, a linguistic term of the type "healthy", "unhealthy", "sexually active", "non-active", "isolated", "non-isolated", etc. A fuzzy "patch" represents a fuzzy "rule", a rule "patch".

[11] See reference item [9]

From a linguistic point of view:

- The Input Space, X, represents a "noun". Similarly, the Output Space, Y, represents another "noun". For instance, "Flu" or "AIDS carrier" could replace X and "Infection", "Fever", "System", "Count" ("Symptom" in short) could replace Y
- The Input Labels ("A"s) and Output Labels ("B"s) represent "adjectives" like "weak", "high", "strong", "normal", "hot", "slow", "normal", "fast", "immune", etc. They also represent "fuzzy sets"
- The "patches" and the "rectangle" represent "statements" linking the Input Space and the Output Space, that is, the nouns and adjectives in the Input Space are linked to the nouns and the adjectives in the Output Space. For instance, "IF X is A_x THEN Y is A_y" may be {IF "the AIDS Carrier" IS "sexually very active" THEN "the probability of spreading Infection" IS "high".}

The sum of all rules (patches, rectangles) helps to approximate the analytically unknown relationship between input and output. We can see this by looking at Figure 2, which shows how the fuzzy function Y = F(X) can be implemented either in hardware or as a software algorithm.

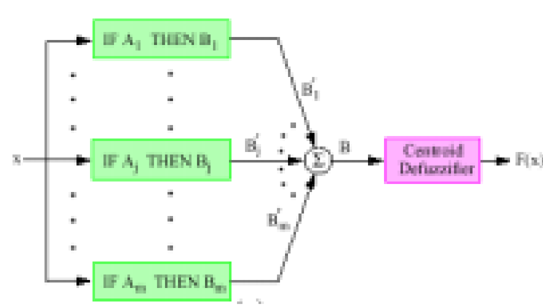

Figure 2 - – "Standard Additive Model"

The "Fuzzy Approximation Theorem" proved in 1990 by Bart Kosko, shows that a fuzzy system can model or approximate any system[12]. Because of this theorem: "In theory we can translate all the equations into rule patches"[11]. In practice this is not always feasible because of the exponential rule explosion due to the level of precision that is required in some applications.

2.4 Mamdani[13] Model vs Sugeno[14] Model

The Mamdani model is more suitable for the use of fuzzy logic as a way of translating the "application domain" expert's experience into a computer algorithm by interviewing the expert and extracting the applicable rule base.

The Mamdani model is useful to model behavioural changes of individuals as a result of the epidemics. This affects how individuals mix.

The Sugeno model is a more suitable tool for adaptively extracting rules from the data itself.

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[12] See Reference items [9], [11], [12], and [13]

[13] A type of fuzzy inference in which the fuzzy sets from the consequent of each rule are combined through the aggregation operator and the resulting fuzzy set is defuzzified to yield the output of the system.

[14] This is a type of fuzzy inference in which the consequent of each rule is a linear combination of the inputs. The output is a weighted linear combination of the consequents. All output membership functions are singleton spikes. The implication method is simply multiplication, and the aggregation operator just includes all of the singletons

2.5 Fuzzy CA/LGCA

Fuzzy CA/LGCA are currently still little studied.

The authors have not yet developed a stand-alone computer program. MATLAB and the Fuzzy Logic Toolbox, a companion product to MATLAB, are being used to generate a Sugeno model from the data. This has proven to be computationally very intensive.

A preliminary Mamdani model is being derived after interviewing some "application domain" experts, that is physicians.

3. Conclusions

A software tool for modelling the spread of epidemics has been presented in [2]. The key feature of this model is that being based on a Lattice Gas Cellular Automaton, it can help identifying spatial characteristics of the epidemics. The model can easily be modified to account for behavioural changes of the population affected by the epidemic. However, this tool cannot deal with imprecise and vague data unless a suitable methodology is added for this purpose. The authors have decided to use fuzzy logic. Preliminary results look promising. The authors plan to apply the combination of LGCA and Fuzzy Logic to model various types of epidemics.

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References

[1] H. Fukś, A. Lawniczak, "Individual based lattice model for the spatial spread of epidemics", 2000, The Fields Institute for Research in Mathematical Sciences Report

[2] B. Di Stefano, H. Fukś, A. Lawniczak, "Object-Oriented Implementation Of CA/LGCA Modelling Applied The Spread Of Epidemics", CCECE'2000, Halifax, Nova Scotia, Canada, May 7-10, 2000

[3] Denis Mollison (Editor), "Epidemic Models: their Structure and Relation to Data", Publications of the Newton Institute, Cambridge University Press 1995, ISBN: 0521475368

[4] Stephen Wolfram, "Statistical Mechanics of Cellular Automata", originally published in Reviews of Modern Physics, volume 55, pages 601-644 (July 1983) – republished in Stephen Wolfram, "Cellular Automata and Complexity – Collected Papers", 1994 Addison-Wesley Publishing Company, ISBN: 0-201-62716-7 (hardcover) or 0-201-62664-0 (paperback)

[5] Thomas C. Quinn and Anthony S. Fauci, "The AIDS Epidemic: Demographic Aspects, Population Biology, and Virus Evolution", 1998 Academic Press

[6] Lotfi A. Zadeh, "Fuzzy Logic = Computing with Words", IEEE Transactions on Fuzzy Systems, Vol. 4, No. 42, May 1996

[7] Lotfi A. Zadeh, "From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions", IEEE Transactions on Circuits and Systems, Vol. 45, No. 1, January 1999, page 109

[8] Bart Kosko, "Fuzzy Thinking – The New Science of Fuzzy Logic", Hyperion/Disney Books, 1993 (ISBN 0-7868-8021-X).

[9] Sanya Mitaim and Bart Kosko, "Adaptive Joint Fuzzy Sets for Function Approximation", Proceedings of the 1997 International Conference on Neural Networks (ICNN-97), pp. 537-542, June 1997

[10] Bart Kosko, Sanya Mitaim, "What is the Best Shape for a Fuzzy Set in Function Approximation?", Proceedings of the 5th IEEE International Conference on Fuzzy Systems (FUZZ-96), pp. 1237-1243, September 1996

[11] B. Kosko, "Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence", Englewood Cliffs, NJ: Prentice Hall, 1992, ISBN 0 13 611 435 0

[12] Bart Kosko, "Fuzzy Engineering, 1/e", 1996 Prentice Hall, (ISBN 0-13-124991-6)

[13] Bart Kosho, "Fuzzy Systems as Universal Approximators", Proceedings of the First IEEE Conference on Fuzzy Systems (FUZZ-92), pages 1153-62, San Diego, March 1992